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Minimum weight buildings design using inequalities method

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Summary

Based on inequalities method and on possibilities of solving by automatic computation mathematical computation model of minimum weight steel structures is presented

KEYWORDS: optimum design, minimum weight, inequalities method, linear algorithm..

1. GENERAL PROBLEMS

Optimization computation due to weight criterion can be done in many ways, according to structure equilibrium equations which are designed function of weight reduction condition formulation. Most important ways are presented below:

1. Establishment, function of a certain cross-section previously chosen, of all possible solutions, determination for each of it of the allowable load and choosing from all "sure" (for which the capable load is bigger than real load) of the one with the smallest one. The procedure is very elaborated and represents an empirical way for obtaining of some economic solutions.

2. The computation is based on choosing the most efficient solution from a variety of solutions, using one of the post-elastic computation methods. Such a possibility is given by the bending moments distribution in plastic domain, in its usual form or in a generalized operation form with mechanical work measures. [1] Using of this method supposes a certain experience in choosing the adjusting way of nodes, bars and kinematic chains equilibration, respectively in choosing of some combinations and constructive constraints which have to be taken in view.

3. Static methods, based on plastic yielding conditions (safety) corresponding to critical cross-sections of the structure, as presented below:

$$-S_{p(i)} \le S_i \le S_{p(i)} \qquad (1)$$

where: $S_{p(i)}$ is the capable effort (plastic) of the critical cross-section "i";

S_i is the effective effort in the critical cross-section "i".

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In the case of bar structures subjected mostly in bending, relation (1) becomes:

$$-M_{p(i)} \leq M_i \leq M_{p(i)} \tag{2}$$

Choosing a statically determined system and writing the bending moments M_i as:

$$-M_i = M_{0i} + \lambda \cdot M_{0i}^{'} + \sum_{h=1}^{n} x_h \cdot m_{0i}^{h} \tag{3}$$

where: M_{0i} - bending moments on the base system, produced by permanent loads;

λ - loading coefficient;

$M_{0i}^{'}$ - bending moments in the base system by $\lambda = 1$;

x_h - statically undetermined values;

m_{0i}^{h} - bending moments produced in the base system by each $x_h = 1$.

The limit state computation is reduced to a linear algorithm problem consisting in increasing at maximum the loading factor λ in obtained relations by replacing the equations (3) and (2). By utilizing bending moment diagrams in equilibrium with external loads, disposed on base systems, judiciously choused, and which can differ from a load case to another, the plastic yielding conditions become $[7]$:

$$-M_p \leq M_p^0 + \sum x_i \cdot m_i \leq M_p \tag{4}$$

where (for a critical cross-section "i"):

M_p - plastic moment of the cross-section;

M_p^0 - bending moment produced by several loads on a certain base system;

x_i - proportionality coefficients (statically undetermined values);

m_i - bending moments from auto equilibrium diagrams.

Using unknown factors decomposing or axes translations, are obtained resolvable formulations by simple procedure, due to which is determined the minimum of weight function.

4. The method based on using the elementary mechanisms combination and on the minimum weight solution theory elaborated by J.Foulkes and B.G.Neal.

5. The method proposed by J.Heyman and W.Prager, based on the general conditions of the limit state computation and on the minimum weight design theories and following the static way, in a two cycles of solving, each cycle consisting of two stages.

Generally, additionally to general hypothesis of the post elastic computation domain, at minimum weight structures design are also taken into consideration some supplementary hypothesis:

a) There exist an infinite variety of cross-sections (rolled steel shapes or composed cross-sections);

b) It is known the variation law on the weight per unit length of elements (q), due to plastic strength modulus (W_p). If there are graphically represented the pairs of values of (q, W_p) for the usual :I" rolled steel shape, is obtained the curve presented in figure 1 (approximated as being a continuous curve), which can be expressed by an exponential relation as :

$$q = k \cdot W_p^{\alpha} \tag{5}$$

or:

$$q = k \left(\frac{M_p}{\sigma_c} \right)^{\alpha} = k' \cdot M_p^{\alpha} \tag{6}$$

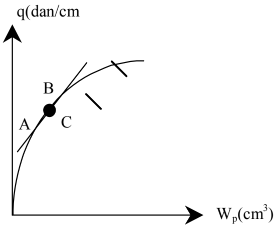

Fig. 1

The coefficients k and α vary function of the cross-section type.

Since for a certain structure the used cross-sections don't vary generally between very high limits, the curve (q, W_p) can be approximated with a polygonal diagram. For example, if the values of plastic module corresponding to points A and B from the figure 1 have a 1:2 ratio, the error coming from the approximation of the curve on the specified portion with a straight line, is only of 1%. Due to this assumption, the weight per unit length of the element can be written as a linear function:

$$q = a + b \cdot M_p \tag{7}$$

The total weight of the structure is:

$$Q = \sum_{i=1}^{n} q_i \cdot l_i = \sum_{i=1}^{n} \left(a + b \cdot M_{p(i)} \right) \cdot l_i = a \cdot \sum_{i=1}^{n} l_i + b \cdot \sum_{i=1}^{n} M_{p(i)} \cdot l_i \tag{8}$$

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where l_i represents the lengths of the bars, $M_{p(i)}$ are the plastic bending moments of the bars cross-sections, and the number „n" is the number of bars having different values of plastic bending moments.

Because $a \cdot \sum_{i=1}^{n} l_i$ and b are constant, results that the determination of the minimum weight is reduced to determination of the minimum solution of the equation:

$$X = \sum_{i=1}^{n} M_{p(i)} \cdot l_i \tag{9}$$

called weight function.

Regarding the loads considered for minimum weight structures computation, according to American norms, they will be considerate as follows:

- for the combination consisting of permanent and live loads, they will be multiplied with a unic coefficient 1,7;
- for the combination consisting of permanent loads, live loads and wind action or earthquake action, they will be multiplied with a unic coefficient 1,3.

2. COMPUTATION MODEL BASED ON INEQUALITIES METHOD

Adaptation of the inequalities method for optimization computation

Inequalities method, usually used as determination way of loading limit factor can be adapted for a structure weight optimization computation, representing some important advantages, as simple and direct way of writing the constraining relations (especially of plastic yielding conditions), and the fact that it can be taken into consideration in the optimization computation of axial force influence, fact that can influence a lot the conceiving and behavior of certain structures categories.

Adaptation of inequalities method for weight optimization computation requires two important elements:

a) Taken in consideration as unknowns – in the relations that express plastic yielding conditions – of the plastic moments and introducing in these relations of the loading factor with imposed values by real loads acting on the structure.
b) Joining to these relations the weight function (linear or nonlinear) which has to be optimized.

The relations which compose the mathematic model for optimum design in this way are the following:

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1. Static equilibrium equations:

a) for loaded bars (figure 2.a):

$$M_s^{(k)} \cdot b_k + M_c^{(k)} \cdot l_k + M_d^{(k)} \cdot a_k = \lambda_k \cdot a_k \cdot b_k \tag{10}$$

where: k=1,2,…b (*b* being the number of loaded bars over the length).

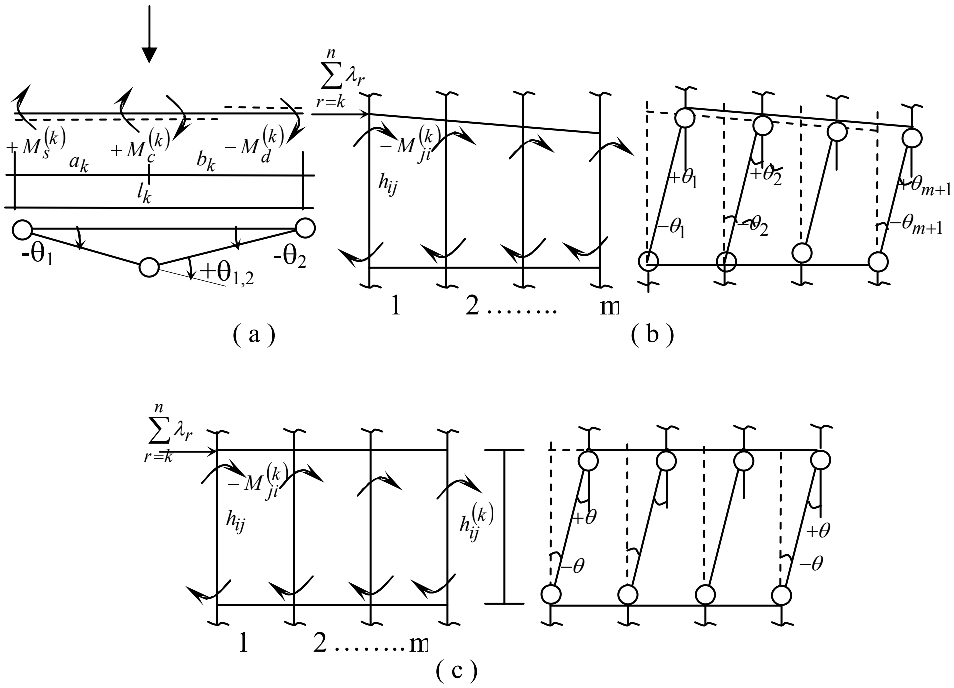

Fig. 2

b) for kinematic chains:

-unequal columns over the level (figure 2.b):

$$\sum_{i=1}^{s} \left(\frac{M_j^{(i)} + M_s^{(i)}}{h_i} \right) = H_k \tag{11}$$

where: i=1,2…s (s being the number of the columns from a level);
k=1,2…m (m being the number of levels of the structure);
hi = column height;
Hk = sliding load for a level.

- unequal columns over the level (figure 2.c):

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$$\sum_{i=1}^{s}\left(M_j^{(i)}+M_s^{(i)}\right)=h_k\cdot H_k \tag{12}$$

h_k level height.

c) for nodes:

$$\sum\left[\pm M_{ij}^{(k)}\right]=0 \tag{13}$$

k=1,2…n (n being the number of nodes).

2. Relations which express plastic yielding conditions:

$$-M_{p(i)}\leq M_i\leq M_{p(i)} \tag{14}$$

where: $M_{p(i)}$ is the plastic moment of the critical cross-section"i" (the unknowns of the problem);

M_i is the effective bending moment in the critical cross-section "i", i=1,2…c (c being the number of critical cross-sections).

Totally are written a number of (e) statically equilibrium relations (e=b+m+n) and a number of (2c) inequalities – plastic yielding conditions.

3. Weight function, which has the usual form:

$$X=\sum_{i=1}^{p}l_i\cdot M_{p(i)}^{\alpha} \tag{15}$$

i=1,2…n (n being the established number of different plastic bending moments of the structure).

In a matrix form, the relations can be written as:

- statically equilibrium relations:

$$[B]\cdot\{\mathrm{M}\}=\{\bar{\lambda}\} \tag{16}$$

- plastic yielding conditions:

$$-\{M_p\}\leq\{\mathrm{M}\}\leq\{M_p\} \tag{17}$$

- weight function:

$$X=\{C\}^T\cdot\{M_p\} \tag{18}$$

where:

$[B]$ is the matrix of coefficients from the statically equilibrium relations;

$\{M\}$ vector of bending moments in the critical cross-sections;

$\{\bar{\lambda}\}$ is the vector of constants from statically equilibrium;

$\{M_p\}$ is the vector of plastic moments in critical cross-sections;

$\{C\}$ is the vector of plastic moments coefficients from the weight equation.

Removing from the plastic yielding conditions a number of (e) bending moments by their replacing with the values obtained in the (e) statically equilibrium equations, function of the other (c-e) bending moments, will result (2c) inequalities with [p+(c-e)] variables, [p necessary plastic bending moments and (c-e) bending moments in the critical cross-sections, as:

$$[A]\cdot\{M\}\geq\{\lambda\} \tag{19}$$

Some inequalities will be eliminated, obtaining a reduced number of constraining conditions which, together with the weight function, compose the relations of programming problem for minimum weight determination.

From the constraints number reducing problem, the most important are the following:

a) Imposing – constructively taking – of some ratios between necessary plastic moments, meaning:

$$M_{p(i)} \gtrless M_{p(k)} \tag{20}$$

which will have as effect the decreasing of the unknowns number of the optimization problem and also will eliminate some constraining relations referring to plastic joints appearance possibility on each bar in nodes.

b) "Selection" of inequalities meaning eliminating the least restrictive relations (which are satisfied including the remained inequalities).

c) Partially or totally knowing the shape of failure bending moments distribution (based on static and loading schemes), which makes possible to write – for critical cross-sections where is certainly known the sign of bending moment – only some simple inequalities, instead of double ones which usually appear in plastic yielding conditions:

$$-M_{p(i)}\leq M_i \tag{21}$$

or:

$$M_i\leq M_{p(i)} \tag{22}$$

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In case of a linear weight function, **simplex method** can be used for solving, when is necessary the transformation of constraints inequations in equalities relations, by introducing of some compensation variables $\overline{M}$, so the matrix relation [19] becomes (eventually after elimination of some constraints conditions):

$$[A]\cdot\{M\}-[E]\cdot\{\overline{M}\}=\{\lambda\} \tag{23}$$

where: $\{\overline{M}\}$ is the vector of compensation variables;

$[E]$ is the unit matrix.

Or, can be used the extended form of the problem, by introducing of some auxiliary variables $M^{'}$:

$$[A]\cdot\{M\}-[E]\cdot\{\overline{M}\}+[E]\cdot\{M^{'}\}=\{\lambda\} \tag{24}$$

or:

$$[A]\cdot\{M\}+[E]\cdot\{M^{*}\}=\{\lambda\} \tag{25}$$

where:

$$\{M^{*}\}=\{M^{'}\}-\{\overline{M}\} \tag{26}$$

in this case, is necessary to respect the negativity conditions for all three variables categories:

$$M,\overline{M},M^{'}\geq 0 \tag{27}$$

and the extended weight function (the lower bound of the weight function X) is:

$$X^{*}=\{C\}^{T}\cdot\{M_{p}\}+\{0\}^{T}\cdot\{M_{i}\}+\{\mu\}^{T}\cdot\{M^{*}\} \tag{28}$$

where: $\{M_{p}\}$ is the vector of necessary plastic bending moments, which makes the minimum weight structure;

$\{M_{i}\}$ is the vector of remain bending moments;

$\{M^{*}\}$ is the vector of auxiliary compensation variables.

The simplex solution contains also the values of “p” necessary plastic bending moments, and the values of remain bending moments in (c-e) critical cross-sections; the other (e) values of bending moments are determined with statically equilibrium equations, so it is possible a complete statically analyze for checking the plastic yielding conditions fulfill, and also the failure mechanism establishment.

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In case of some big dimensions, the problem will be solved with a linear or nonlinear computer program.

3. COMPUTATION EXAMPLE

Determination of minimum weight solution for the frame in the figure 3.a.

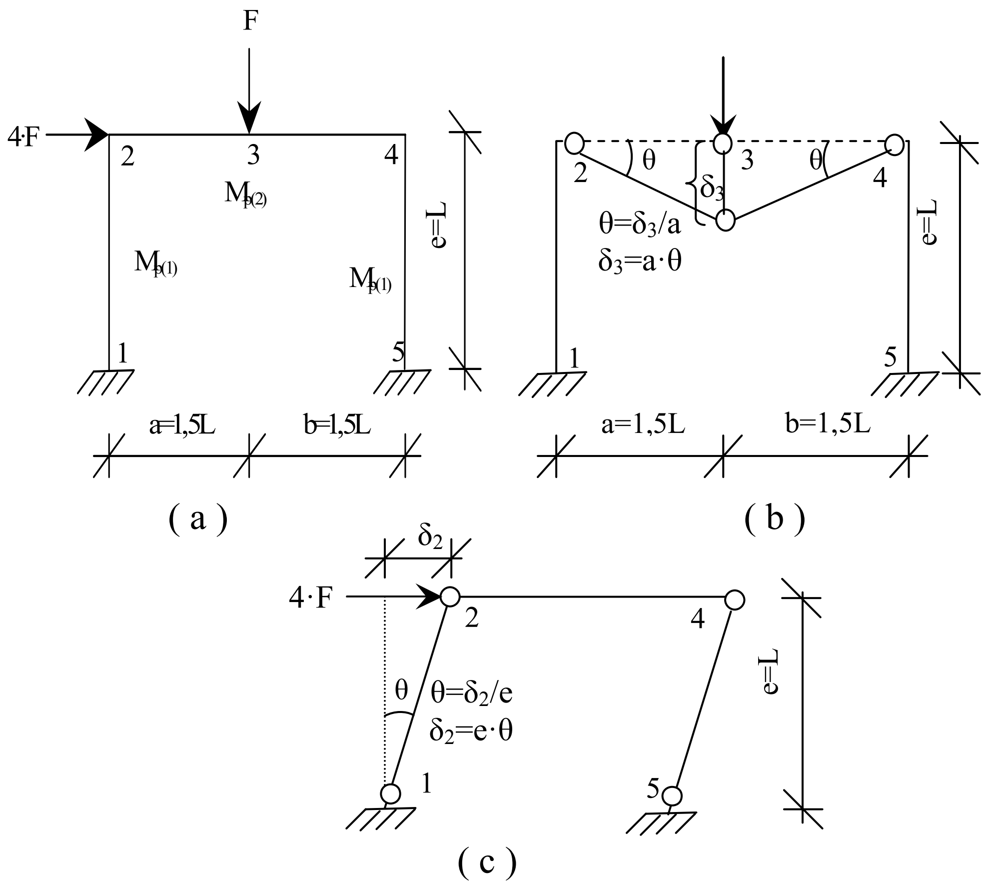

Fig. 3

It is considered: $F = 1$; $L = 1$.

It will be noted: $M_{p(1)} = Y_1$ şi $M_{p(2)} = Y_2$ which are the necessary plastic bending moments for the columns and beams.

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Will be noted the bending moments in the five critical cross-sections **(1,2,3,4,5)** of the frame, as**:** $M_3 = Y_3$; $M_4 = Y_4$; $M_5 = Y_5$; $M_1 = Y_6$; $M_2 = Y_7$

The relations which compose the mathematical model of the minimum weight frame design problem by **inequalities method** are the following:

1) Statically equilibrium relations:

a) bar equilibrium (figure 3.b):

$$M_2 \cdot \theta + M_3 \cdot \theta + M_3 \cdot \theta + M_4 \cdot \theta = F \cdot \delta_1$$

or: $M_2 + M_3 + M_3 + M_4 = F \cdot a$

having the established notations:

$$Y_7 + Y_3 + Y_3 + Y_4 = F \cdot 1{,}5 \cdot L$$

or $Y_7 + 2 \cdot Y_3 + Y_4 = 1{,}5$

b) displacement equilibrium (figure 3.c):

$$M_1 \cdot \theta + M_2 \cdot \theta + M_4 \cdot \theta + M_5 \cdot \theta = 4 \cdot F \cdot \delta_2$$

or: $M_1 + M_2 + M_4 + M_5 = 4 \cdot F \cdot e$

having the established notations:

$$Y_6 + Y_7 + Y_4 + Y_5 = 4 \cdot F \cdot L$$

$$Y_6 + Y_7 + Y_4 + Y_5 = 4$$

2) Plastic yielding conditions:

$$-Y_1 \le Y_6 \le Y_1; -Y_1 \le Y_7 \le Y_1;$$

$$-Y_2 \le Y_7 \le Y_2; -Y_2 \le Y_3 \le Y_2; -Y_2 \le Y_4 \le Y_2; -Y_1 \le Y_4 \le Y_1;\ -Y_1 \le Y_5 \le Y_1$$

These relations can also be written:

$$Y_1+Y_6 \ge 0;\ Y_1+Y_7 \ge 0;\ Y_2+Y_7 \ge 0;\ Y_2+Y_3 \ge 0;\ Y_2+Y_4 \ge 0;\ Y_1+Y_4 \ge 0;\ Y_1+Y_5 \ge$$

respectively:

$$Y_1-Y_6 \ge 0;\ Y_1-Y_7 \ge 0;\ Y_2-Y_7 \ge 0;\ Y_2-Y_3 \ge 0;\ Y_2-Y_4 \ge 0;\ Y_1-Y_4 \ge 0;\ Y_1-Y_5 \ge 0$$

3) Weight function:

$$X = \sum_{1}^{n} l_i \cdot M_{p(i)}$$

so: $X=2\cdot M_{p(1)}+3\cdot M_{p(2)}+0\cdot M_3+0\cdot M_4+0\cdot M_5+0\cdot M_1+0\cdot M_2$

or: $X=2\cdot Y_1+3\cdot Y_2+0\cdot Y_3+0\cdot Y_4+0\cdot Y_5+0\cdot Y_6+0\cdot Y_7$

Using a usual computation program for solving linear problems, will be obtained the following results:

- plastic bending moments values:

$Y_1=M_{p(1)}=1.625$; $Y_2=M_{p(2)}=0.375$

- value of moments from critically sections:

$Y_3=M_3=0.375$; $Y_4=M_4=0.375$; $Y_5=M_5=1.625$; $Y_6=M_1=1.625$; $Y_7=M_2=0.375$

- value of the weight function:

X=4.375

Knowing the values of plastic bending moments on the columns and beams will be established the cross-section of bars:

a) the column:

$$M_{p(1)} = \sigma_c \cdot W_{column} \quad so: W_{column} = \frac{M_{p(1)}}{\sigma_c}$$

or: $$W_{stalp} = \frac{b_{column} \cdot h_{column}^2}{4}$$

Will be imposed: $h_{column} = 1,5 \cdot b_{column}$

and results : $$\frac{b_{column} \cdot (1,5 \cdot b_{column})^2}{4} = \frac{M_{p(1)}}{\sigma_c}$$

where $$b_{column} = \sqrt[3]{\frac{4 \cdot M_{p(1)}}{2,25 \cdot \sigma_c}}$$

$$h_{stalp} = 1,5 \cdot \sqrt[3]{\frac{4 \cdot M_{p(1)}}{2,25 \cdot \sigma_c}}$$

b) the beam:

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$$M_{p(2)} = \sigma_c \cdot W_{beam} \quad deci : W_{beam} = \frac{M_{p(2)}}{\sigma_c}$$

or:

$$W_{beam} = \frac{b_{beam} \cdot h_{beam}^2}{4}$$

It will be imposed: $h_{beam} = 1{,}5 \cdot b_{beam}$

and results:

$$\frac{b_{beam} \cdot (1{,}5 \cdot b_{beam})^2}{4} = \frac{M_{p(2)}}{\sigma_c}$$

where:

$$b_{beam} = \sqrt[3]{\frac{4 \cdot M_{p(2)}}{2{,}25 \cdot \sigma_c}}$$

$$h_{beam} = 1{,}5 \cdot \sqrt[3]{\frac{4 \cdot M_{p(2)}}{2{,}25 \cdot \sigma_c}}$$

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